Real space sextics and their tritangents

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joint work with:

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- Bernd Sturmfels (MPI Leipzig)
- Mahsa Sayyary (MPI Leipzig)

New York, 20 July 2018

I. An old debt.

Space sextics and their tritangents

Definition

- A *space sextic* is a smooth curve $C \subseteq \mathbb{P}^3$ of the form

  $C = V(f) \cap V(g)$ with $f, g \in \mathbb{C}[x_0, \ldots, x_3]$ homog. of degrees $2, 3$ resp.

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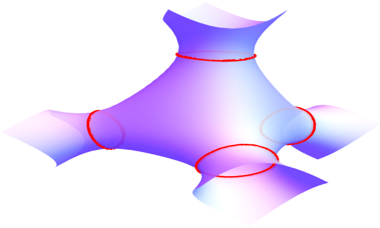

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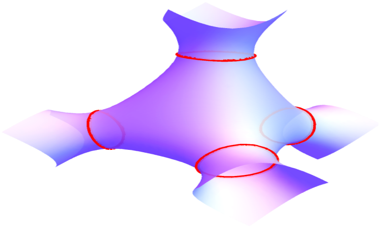
- A *tritangent* is a plane $H \subseteq \mathbb{P}^3$ such that

  $H = V(\ell)$ with $\ell \in \mathbb{C}[x_0, \ldots, x_3]$ of degree 1 and $C \cap H = 3$ double pts.



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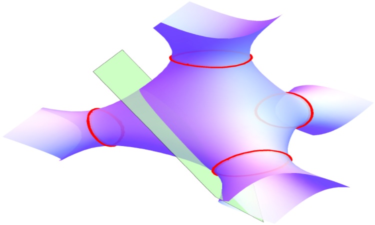

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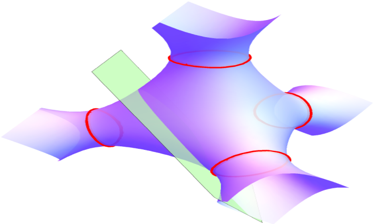
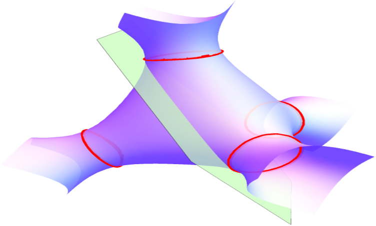
  $H = V(\ell)$ with $\ell \in \mathbb{R}[x_0, \dots, x_3]$ of degree 1 and $C \cap H = 3$ double pts.
- We call H totally real, if $C \cap H$ is real.





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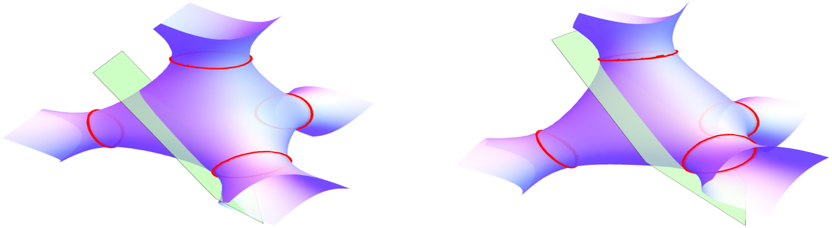

Questions (Clebsch 1864)

- How many tritangents does a space sextic have?
- How many can be real?
- How many can be totally real?

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Answers (Coble 1961)

- There are always 120 complex tritangents.

Space sextics and their tritangents

Questions (Clebsch 1864)

- How many tritangents does a space sextic have?
- How many can be real?
- How many can be totally real?

Answers (Coble 1961, Krasnov 1996)

- There are always 120 complex tritangents.
- The number of real tritangents depends solely on the number of real connected components

# conn. comp.	1	2	3	4	5
real	8	16	32	64	120

Space sextics and their tritangents

Questions (Clebsch 1864)

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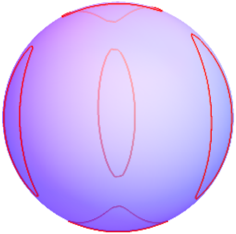
Theorem (Emch 1928)

I have a sextic with 120 totally real trit.



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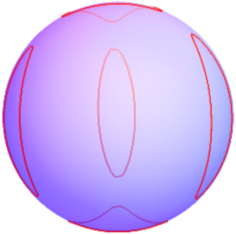
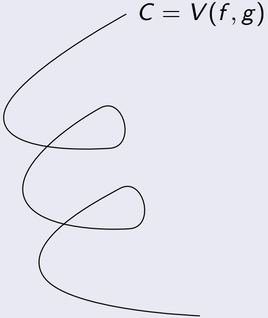


Computing tritangents symbolically

Algorithm

In: $f, g \in \mathbb{C}[x_0, \ldots, x_3]$ defining equations of a space sextic



Out: $\mathcal{I}_C \subseteq \mathbb{C}[u_0, \ldots, u_3]$ with $V(\mathcal{I}_C) = \{\text{tritangent equations}\}$

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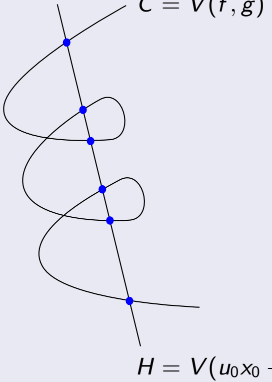

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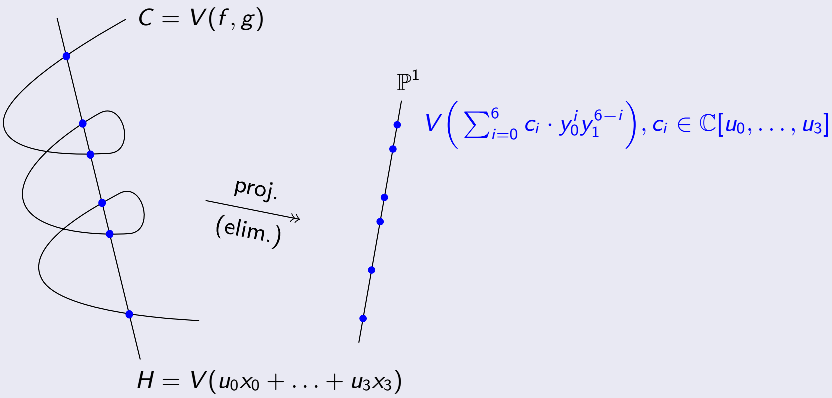

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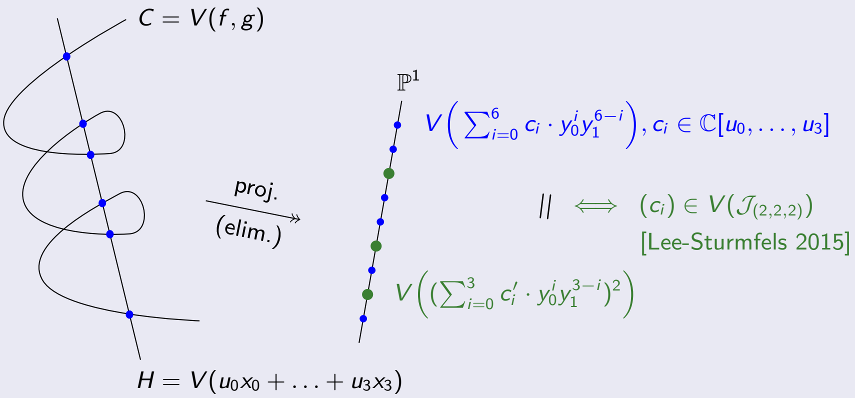

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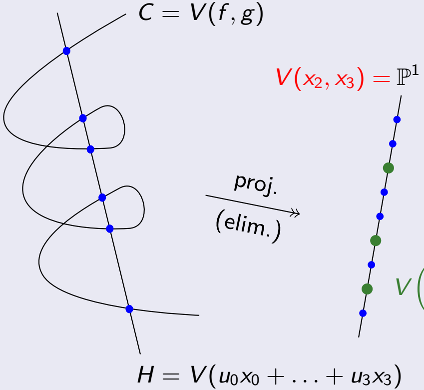

$$\mathrm{Res}_{x_2}\left(f|_{x_3=\frac{u_0x_0+\ldots+u_2x_2}{u_3}}, g|_{x_3=\ldots}\right)$$

$$||$$

$$V\left(\sum_{i=0}^{6} c_i \cdot y_0^i y_1^{6-i}\right), c_i \in \mathbb{C}[u_0, \ldots, u_3]$$

$$|| \iff (c_i) \in V(\mathcal{J}_{(2,2,2)})$$

[Lee-Sturmfels 2015]

$$V\left(\left(\sum_{i=0}^{3} c_i' \cdot y_0^i y_1^{3-i}\right)^2\right)$$

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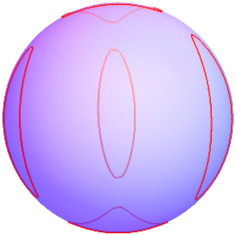

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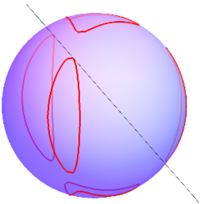
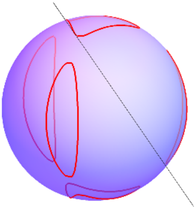
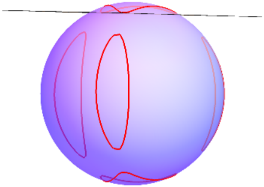
Emch's curve only features 108 totally real tritangents.



$2^3 \cdot \binom{5}{3}$ tritangents of type 111



$3 \cdot 4 + 2 \cdot 9$ tritangents of type $1^2 1$



$2 \cdot 2$ tritangents of type 1^3

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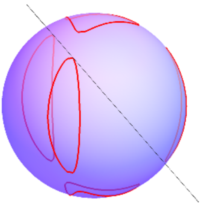
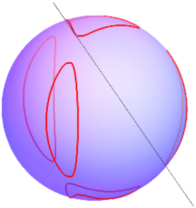


$2^3 \cdot \binom{5}{3}$ tritangents of type 111



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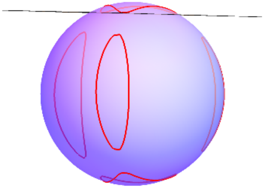

$2 \cdot 2$ tritangents of type 1^3

Data and scripts available at https://software.mis.mpg.de

II. Paying the tab.

Space sextics from del Pezzo surfaces of degree one

$\mathcal{P} \subseteq \mathbb{P}^2$

\

8 distinct pts in general position

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\ 8 distinct pts in general position

$$\dim \left\{ \begin{array}{c} \text{cubics singly} \\ \text{through } \mathcal{P} \end{array} \right\} = \binom{3+3-1}{3} - 8 \cdot 1$$

$$\Cup$$

$$s, t$$

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$$s^3, s^2t, st^2, t^3, sw, tw, r$$

Space sextics from del Pezzo surfaces of degree one

$$\mathcal{P} \subseteq \mathbb{P}^2 \overset{}{\underset{z \mapsto (s(z):t(z):w(z):r(z))}{\dashrightarrow}} \mathbb{P}(1:1:2:3) \supseteq \mathrm{Bl}_{\mathcal{P}}\,\mathbb{P}^2$$

$\mathcal{P}$: 8 distinct pts in general position

$$\dim \left\{ \begin{array}{c} \text{cubics singly} \\ \text{through } \mathcal{P} \end{array} \right\} = \binom{3+3-1}{3} - 8 \cdot 1$$

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Space sextics from del Pezzo surfaces of degree one

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$\mathcal{P}$: 8 distinct pts in general position

$$\mathrm{Bl}_{\mathcal{P}}\,\mathbb{P}^2 \xrightarrow{\text{gen. } 2:1} \mathbb{P}(1:1:2) \supseteq C', \qquad (s:t:w:r) \mapsto (s:t:w)$$

except on: C'

$$\dim \left\{ \begin{matrix} \text{cubics } \color{red}{\text{singly}} \\ \text{through } \mathcal{P} \end{matrix} \right\} = \binom{3+3-1}{3} - 8 \cdot \color{red}{1}$$

$$\cup\!\!\!\!\!\;| \quad s, t$$

$$\dim \left\{ \begin{matrix} \text{sextics } \color{red}{\text{doubly}} \\ \text{through } \mathcal{P} \end{matrix} \right\} = \binom{3+6-1}{6} - 8 \cdot \color{red}{3}$$

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$$\dim \left\{ \begin{matrix} \text{nonics } \color{red}{\text{triply}} \\ \text{through } \mathcal{P} \end{matrix} \right\} = \binom{3+9-1}{9} - 8 \cdot \color{red}{6}$$

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Space sextics from del Pezzo surfaces of degree one

$\mathcal{P} \subseteq \mathbb{P}^2 \dashrightarrow \mathbb{P}(1:1:2:3) \supseteq \mathrm{Bl}_{\mathcal{P}}\,\mathbb{P}^2$, $\quad z \mapsto (s(z):t(z):w(z):r(z))$

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$s^3, s^2t, st^2, t^3, sw, tw, r$

$\mathrm{Bl}_{\mathcal{P}}\,\mathbb{P}^2 \xrightarrow{\text{gen. } 2:1} \mathbb{P}(1:1:2) \supseteq C'$, $\quad (s:t:w:r) \mapsto (s:t:w)$, except on: C'

$\mathbb{P}(1:1:2) \to \mathbb{P}^3$, $\quad (s:t:w) \mapsto (s^2:st:t^2:w)$

$C' \to C$, $\quad \mathbb{P}^3 \supseteq C$

Space sextics from del Pezzo surfaces of degree one

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$\Cup$ $s^3, s^2t, st^2, t^3, sw, tw, r$

$\mathrm{Bl}_{\mathcal{P}}\,\mathbb{P}^2 \to \mathbb{P}(1:1:2)$, gen. 2:1, $(s:t:w:r) \mapsto (s:t:w)$, except on:

$\mathbb{P}(1:1:2) \supseteq C'$

$\mathbb{P}(1:1:2) \to \mathbb{P}^3$, $(s:t:w) \mapsto (s^2:st:t^2:w)$, $C' \to C$

$\mathbb{P}^3 \supseteq V(x_0x_2 - x_1^2) \supseteq C$

Caveat: $V(x_0x_2 - x_1^2)$

Space sextics from del Pezzo surfaces of degree one

$\mathcal{P} \subseteq \mathbb{P}^2 \dashrightarrow \mathbb{P}(1{:}1{:}2{:}3) \supseteq \mathrm{Bl}_{\mathcal{P}}\,\mathbb{P}^2 \supseteq E_1, \dots, E_8$

$z \mapsto (s(z) : t(z) : w(z) : r(z))$

8 distinct pts in general position

gen. 2:1 $\quad (s:t:w:r) \mapsto (s:t:w)$

except on:

$\mathbb{P}(1{:}1{:}2) \supseteq C'$

$(s:t:w) \mapsto (s^2 : st : t^2 : w)$

$H_1, \dots, H_8 \subseteq \mathbb{P}^3 \supseteq \quad \supseteq C$

Caveat: $V(x_0x_2 - x_1^2)$

Lemma

8 of the tritangents of C are images of exceptional divisors over points $P_i \in \mathcal{P}$. Each is totally real if and only if P_i is real.

Space sextics from del Pezzo surfaces of degree one

Lemma

The remaining 112 *tritangents of* C *are the images of:*

Space sextics from del Pezzo surfaces of degree one

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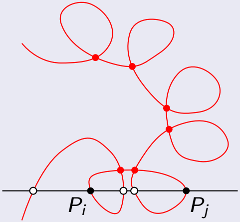
$\binom{8}{2}$: *quintic through* $\mathcal{P}$ *and* $\mathcal{P} \setminus \{P_i, P_j\}$ *doubly*



line through $\{P_i, P_j\}$

Each tritangent is totally real if and only if the white points are real.

Space sextics from del Pezzo surfaces of degree one

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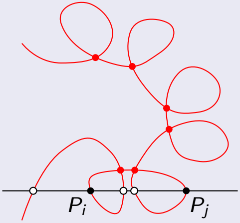
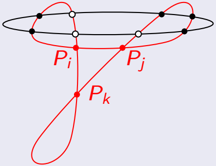


line through $\{P_i, P_j\}$

$\binom{8}{3}$: *conic through* $\mathcal{P} \setminus \{P_i, P_j, P_k\}$



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Space sextics from del Pezzo surfaces of degree one

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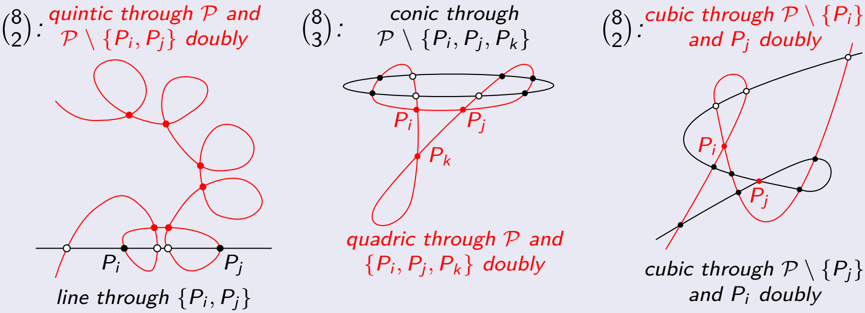

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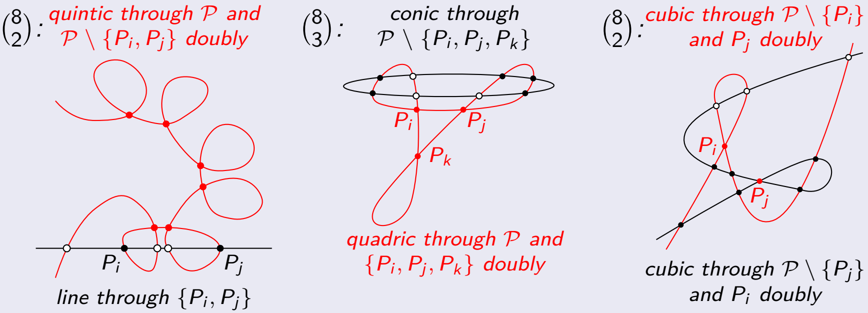

Each tritangent is totally real if and only if the white points are real.

- Computing tritangents of generic space sextics: several minutes.
- Computing tritangents of del Pezzo space sextics: couple of seconds.

Space sextics from del Pezzo surfaces of degree one

Experimental results [KRSS 2018]

# conn. comp.	1	2	3	4	5
complex	120	120	120	120	120
real	8	16	32	64	120
tot. real	[0,8]	[1,15]	[10,32]	[35,64]	[84,**120**]

Space sextics from del Pezzo surfaces of degree one

Experimental results [KRSS + HKSS[1] 2018]

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tot. real	[0,8]	[0,16]	[0,32]	[32,64]	[84,**120**]

1 Hauenstein, Kulkarni, Sertöz, Sherman: *Certifying reality of projections* (2018)

Space sextics from del Pezzo surfaces of degree one

Experimental results [KRSS + HKSS[1] 2018]

# conn. comp.	1	2	3	4	5
complex	120	120	120	120	120
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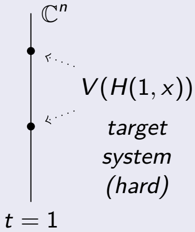
tot. real	[0,8]	[0,16]	[0,32]	[32,64]	[84,**120**]

Remark (Computing tritangents numerically)



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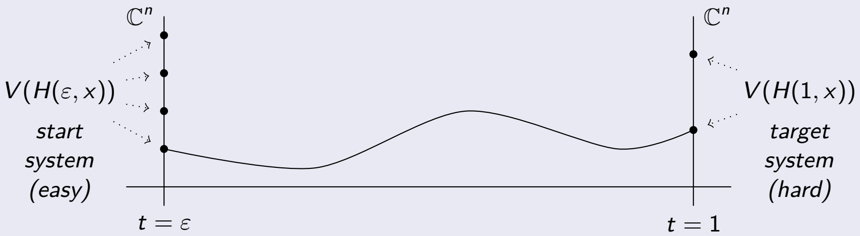

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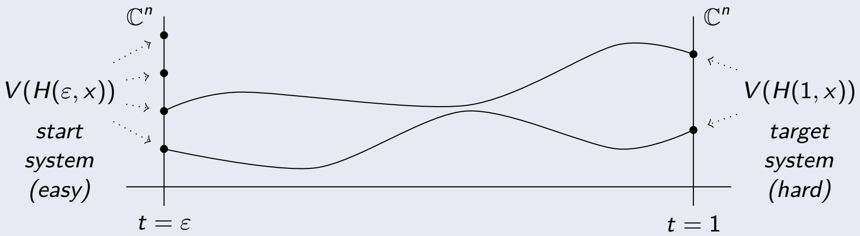

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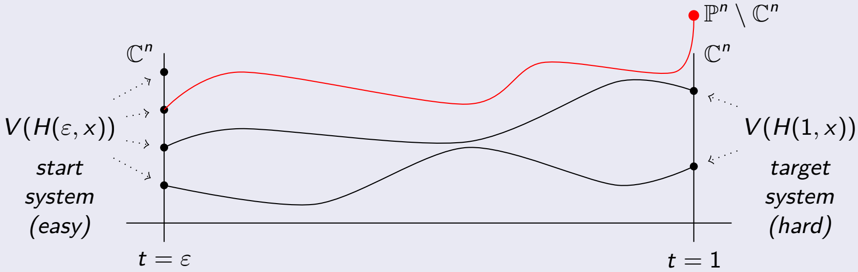

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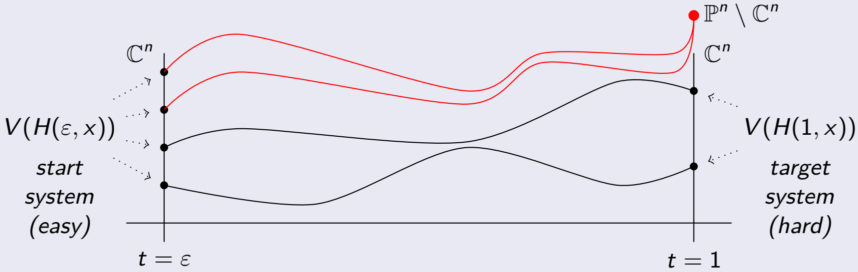

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Remark (Computing tritangents numerically[2])

2 Breiding, Timme: `https://www.juliahomotopycontinuation.org/`

Space sextics from del Pezzo surfaces of degree one

Experimental results [KRSS + HKSS[1] + Kummer[3] 2018]

# conn. comp.	1	2	3	4	5
complex	120	120	120	120	120
real	8	16	32	64	120
tot. real	[0,8]	[0,16]	[0,32]	[32,64]	[84,**120**]
bounds	[0,8]	[0,16]	[0,32]	[32,64]	[80,120]

Theorem (Farkas-Verra[4] 2014, Sertöz[5] 2017)

In the moduli of space sextics, $\mathbb{P}^9 \times \mathbb{P}^{19}$*, the totally real tritangent discriminant is of bi-degree* $(744, 592)$.

3 Kummer: *Totally real theta characteristics* (2018)
4 Farkas, Verra: *The geometry of the moduli space of odd spin curves* (2014)
5 Sertöz: *Enumerative geometry of double spin curves* (2017)

III. Moving on.

What's next?

Experimental results [KRSS + HKSS + Kummer 2018]

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Questions (all of the above)

- Are there space sextics with 80 totally real tritangents?

What's next?

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Questions (all of the above + Caporaso-Sernesi 2003)

- Are there space sextics with 80 totally real tritangents?
- Do the tritangents uniquely determine the space sextic? (true generically)

What's next?

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Current progress

- Symbolic algorithm for reconstruction done. (correct generically)

What's next?

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Questions (all of the above + Caporaso-Sernesi 2003)

- Are there space sextics with 80 totally real tritangents?
- Do the tritangents uniquely determine the space sextic? (true generically)

Current progress

- Symbolic algorithm for reconstruction done. (correct generically)
- Numerical algorithm in work.

What's next?

Numerical Computing in Algebraic Geometry

Summer School, 13-17 August 2018, MPI MIS Leipzig

`https://www.mis.mpg.de/nc2018`

Lecturers:

- Jon Hauenstein (Notre Dame)
- Pablo Parrillo (MIT)
- Nick Vannieuwenhoven (KU Leuven)