Open Non-uniform Cylindrical Algebraic Decomposition

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Context

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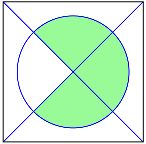
Tarski formula $\leftrightarrow$ **semi-algebraic set**

$$x_1^2 + x_2^2 - 1 < 0 \wedge \left[\begin{array}{c} x_2 + x_1 > 0 \\ \vee \\ x_2 - x_1 < 0 \end{array} \right] \leftrightarrow$$



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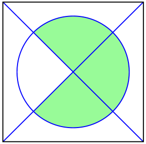

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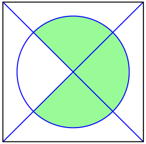

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$$75x + 15y^2 + 16z - 18 > 0 \;\wedge\; -39x - 78yx - 91z - 70 > 0 \;\wedge$$
$$-86x - 44y^2 + 14z - 15 > 0 \;\wedge\; -27xz + 22y - z - 74 > 0 \;\wedge$$
$$55x + 87yz + 45z + 6 > 0 \;\wedge\; 2x^2 + 4y - 13z + 34 > 0$$

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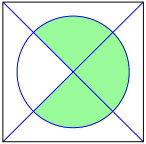


Tarski formulas provide an implicit representation of semi-alg. sets

CAD provides an explicit representation of semi-algebraic sets

This paper's contributions

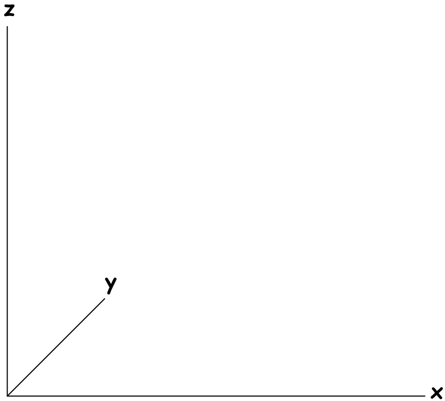
1. introduces Open Non-uniform CAD,
2. provides an algorithm for constructing Open NuCADs from Tarski formulas, and
3. reports results of experiments with an initial implementaton.

Outline

1. Open CAD,
2. Open Non-uniform CAD, and
3. Experimental results.

Open CAD: open cylindrical algebraic cell

An open cylindrical algebraic cell generalizes a box aligned with axes



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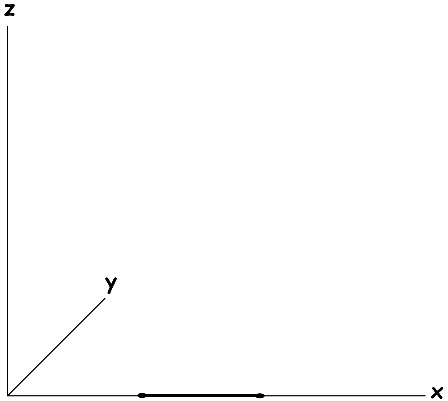

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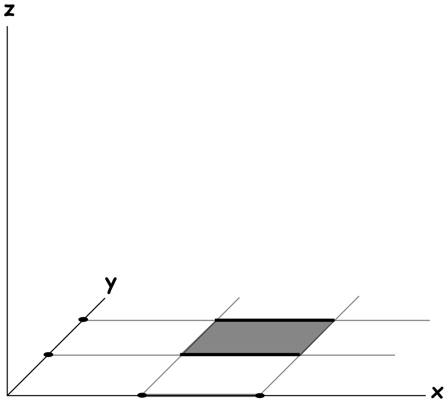

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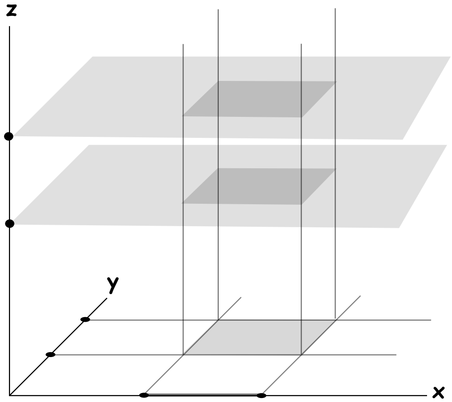

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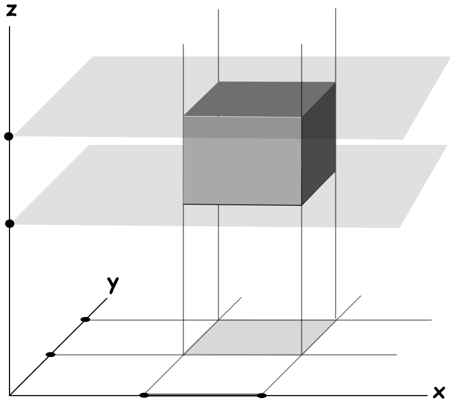

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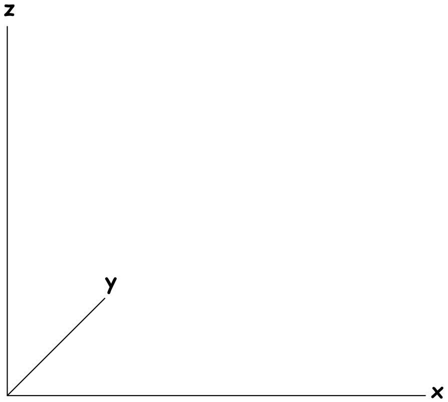

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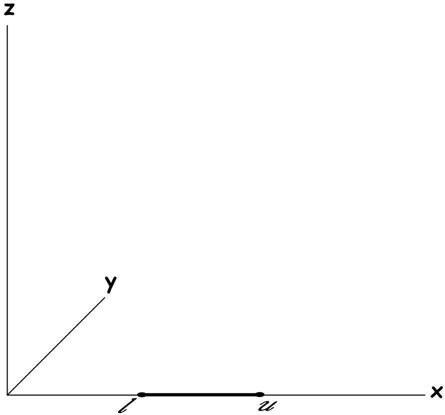

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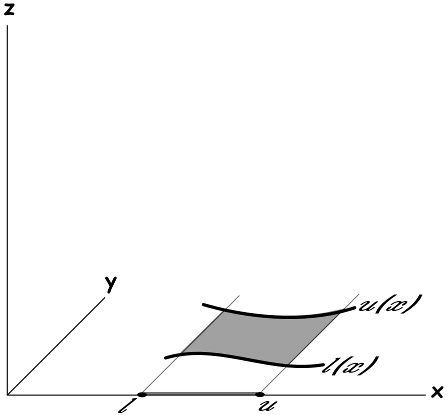

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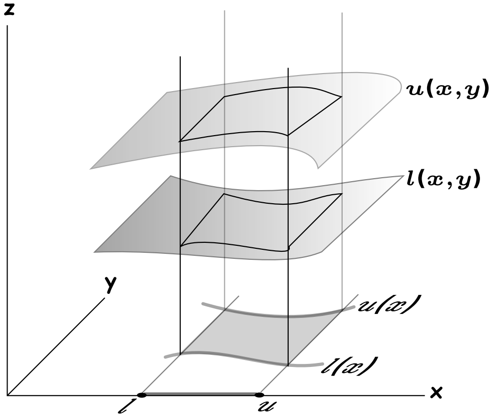

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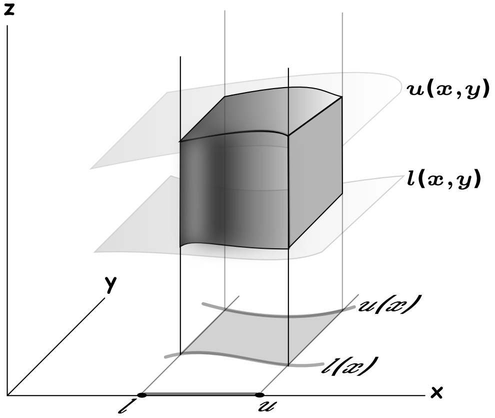

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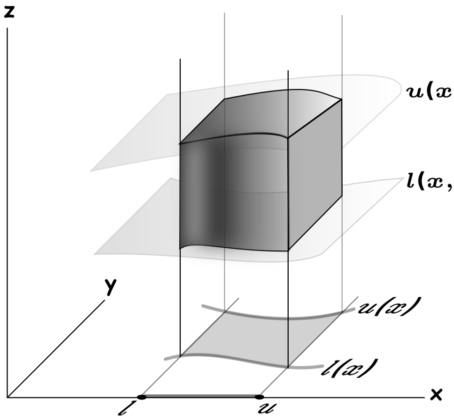

Explicit representation: sample point + upper & lower bound functions

Open CAD: a definition

Definition: An Open CAD is a (weak) decomposition of $\mathbb{R}^n$ into open cylindrical algebraic cells whose arrangement is uniformly cylindrical, meaning that for any two cells c_1, c_2, the projections $\pi_j(c_1), \pi_j(c_2)$ onto $\mathbb{R}^j$ are either identical or disjoint.

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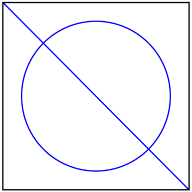
Idea 1: A set A of polynomials in $x_1, \ldots, x_n$ (weakly) decomposes $\mathbb{R}^n$ in a natural way — into connected regions in which $\forall p \in A$, $p \neq 0$

$$A = \{x_1^2 + x_2^2 - 1, x_1 + x_2\} \quad \longrightarrow$$



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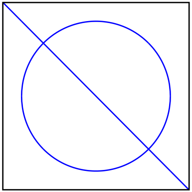
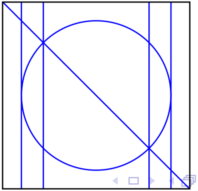


Idea 2: With the right set P of "lower level" polynomials, the natural decomposition defined by $P \cup A$ is an Open CAD.

$$\underbrace{\{x_1 + 1, x_1 - 1, 2x_1^2 - 1\}}_{P} \cup A \quad \longrightarrow$$



Open CAD: an algorithm

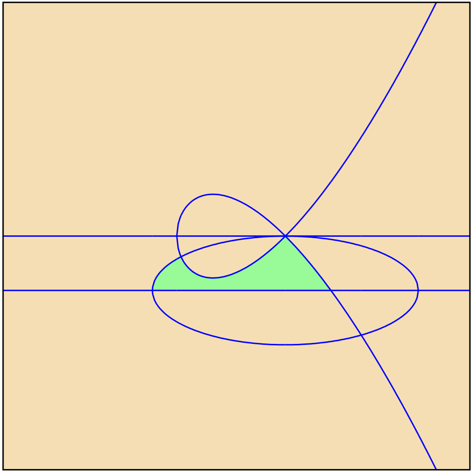
Input: F, a Tarski formula in variables $x_1 \ldots, x_n$
Output: D, an Open CAD of $\mathbb{R}^n$ representing the set defined by F

Step 1: $A \leftarrow$ polynomials in F

Step 2: $P \leftarrow$ projection of A

Step 3: $D \leftarrow$ a list of the open cylindrical cells data structures that explicitly define the CAD given by the natural decomposition of $P \cup A$

Example — constructing an open CAD



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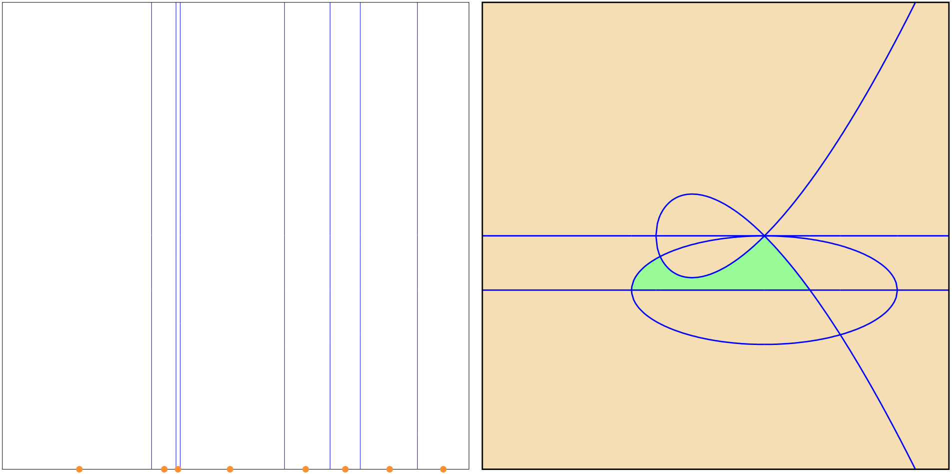

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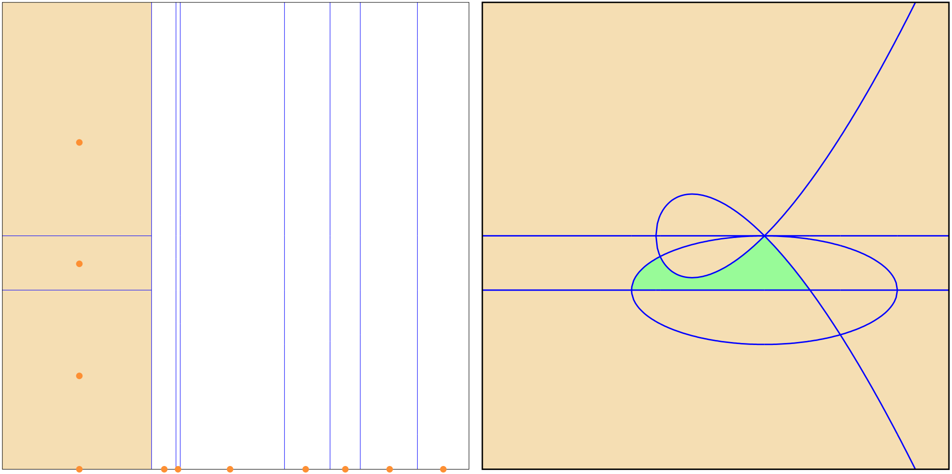

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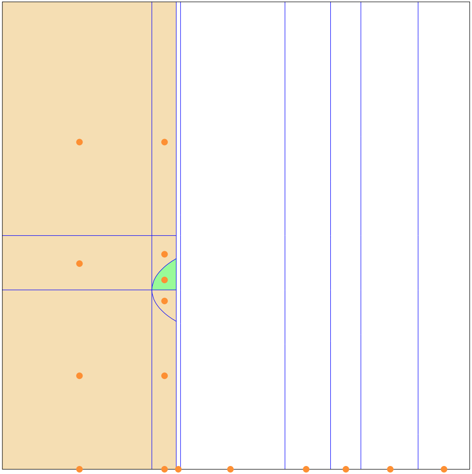

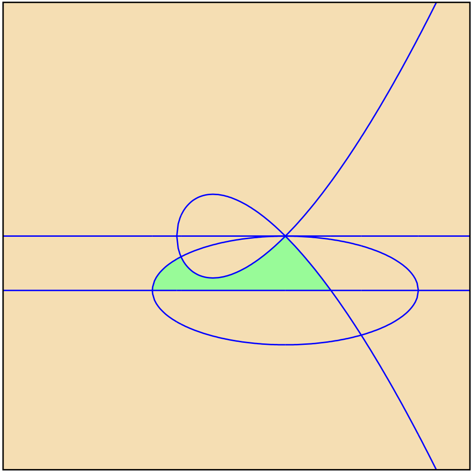

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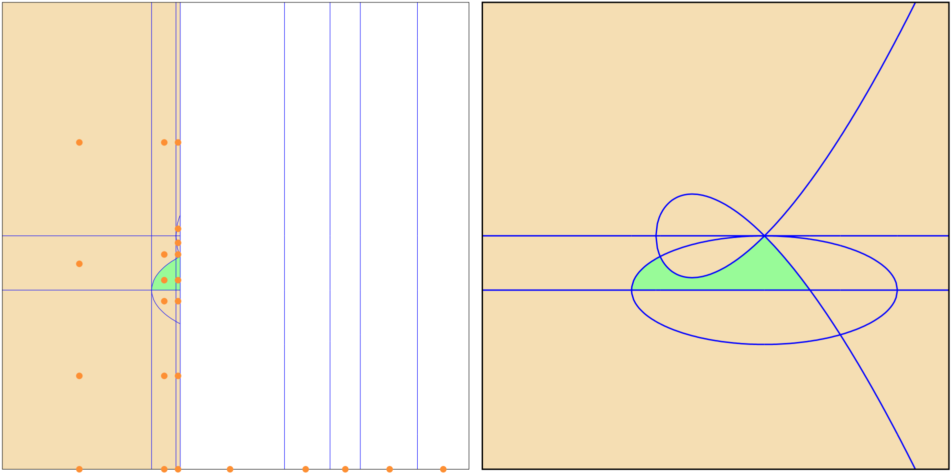

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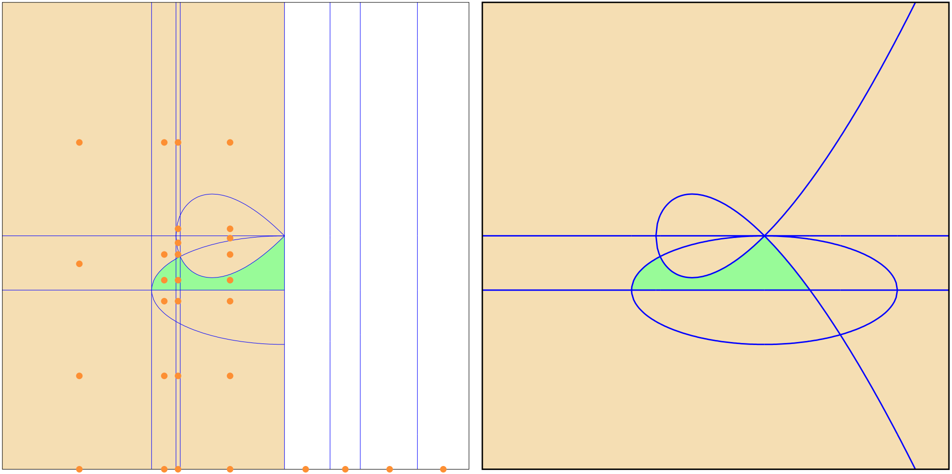

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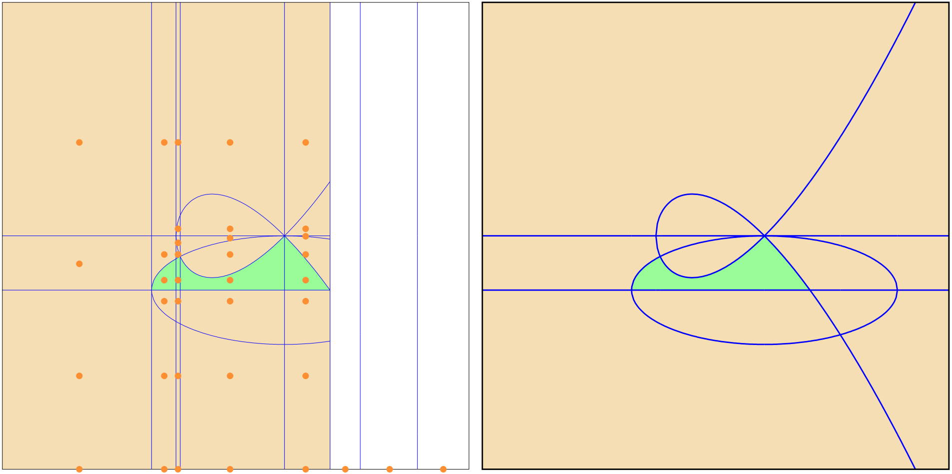

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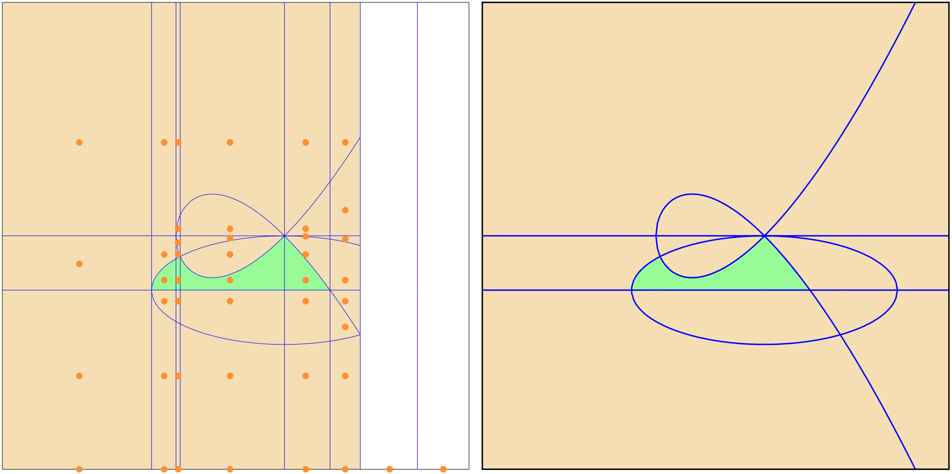

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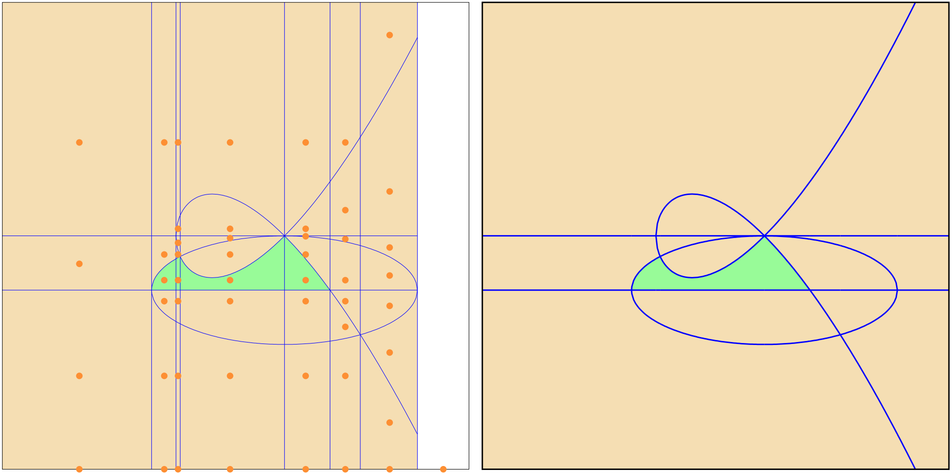

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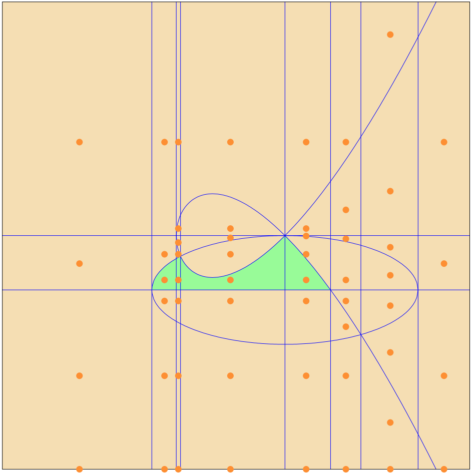

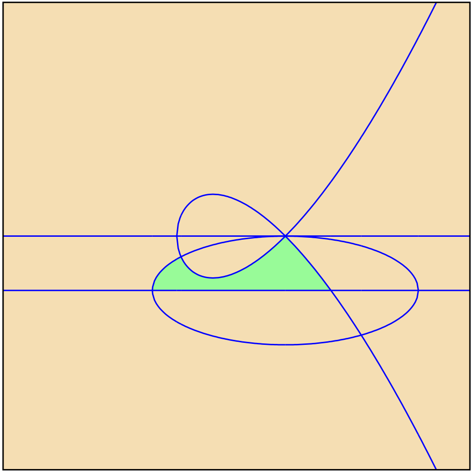

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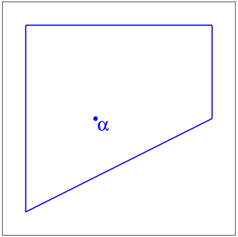

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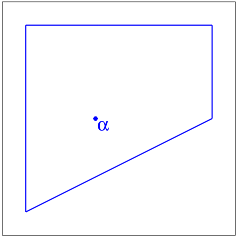

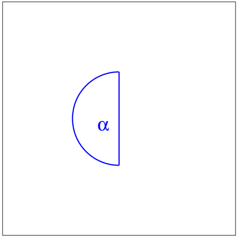

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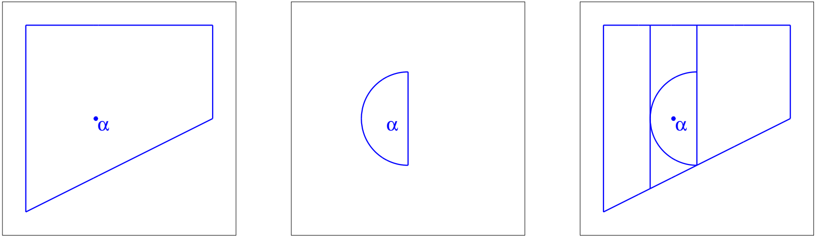

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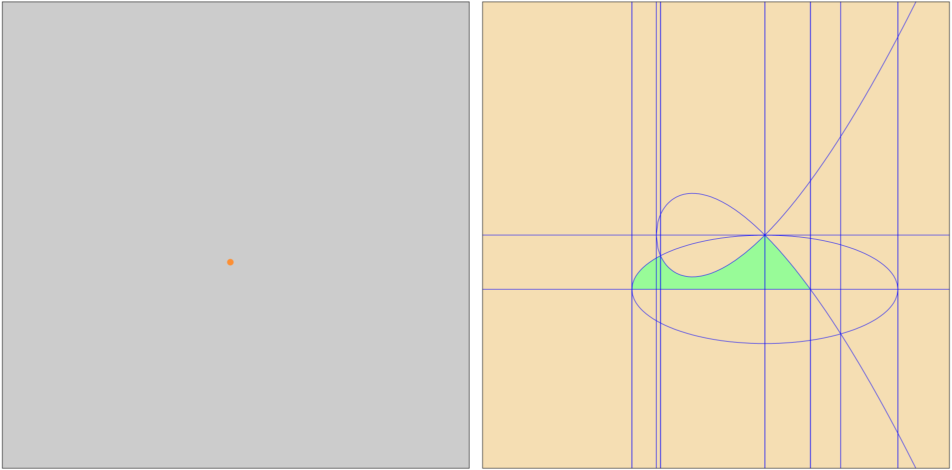

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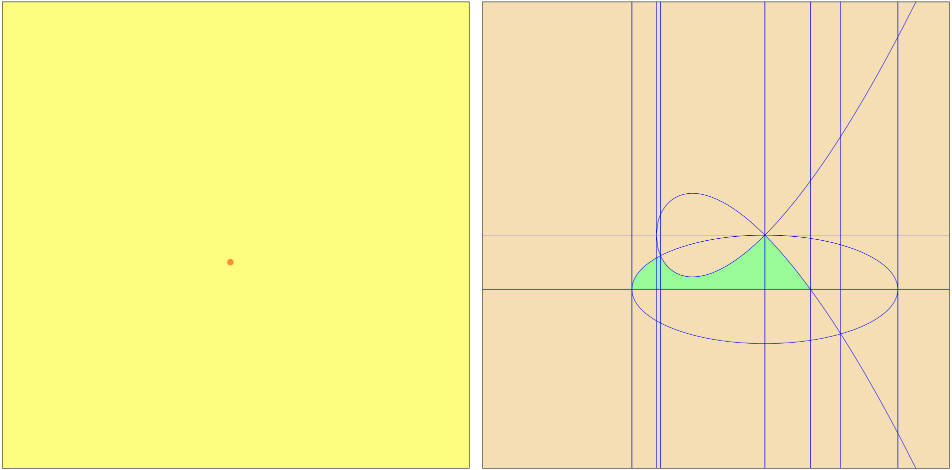

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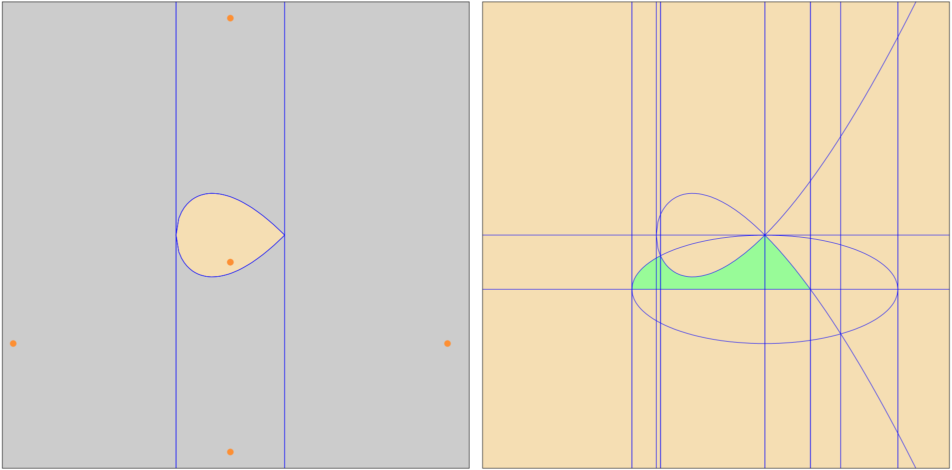

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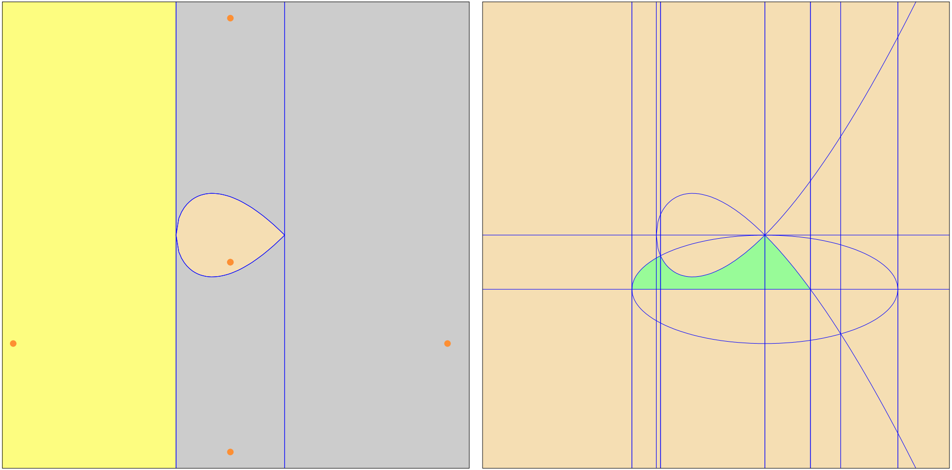

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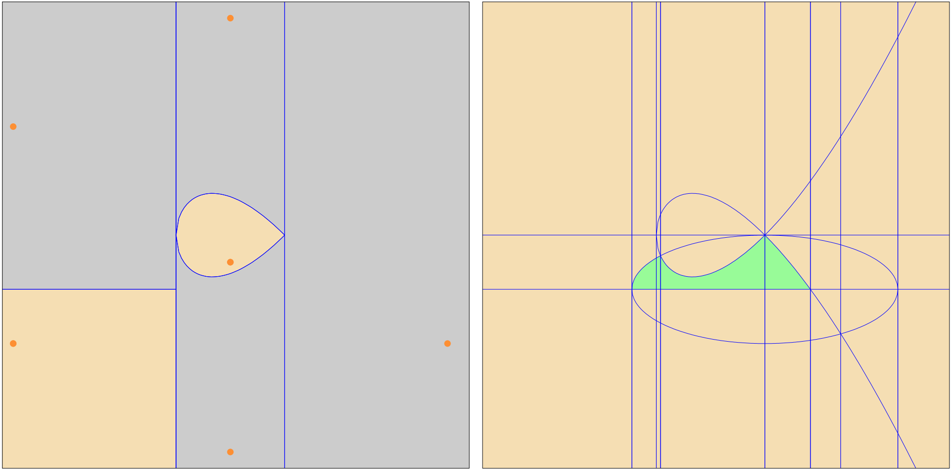

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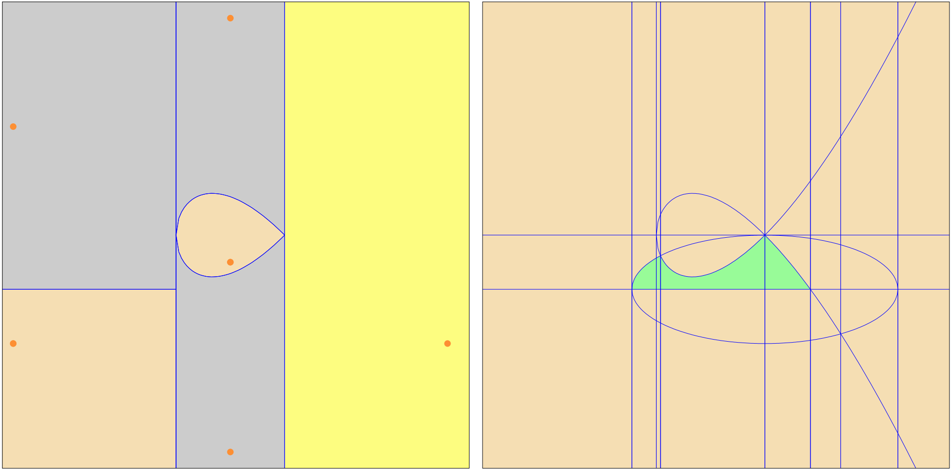

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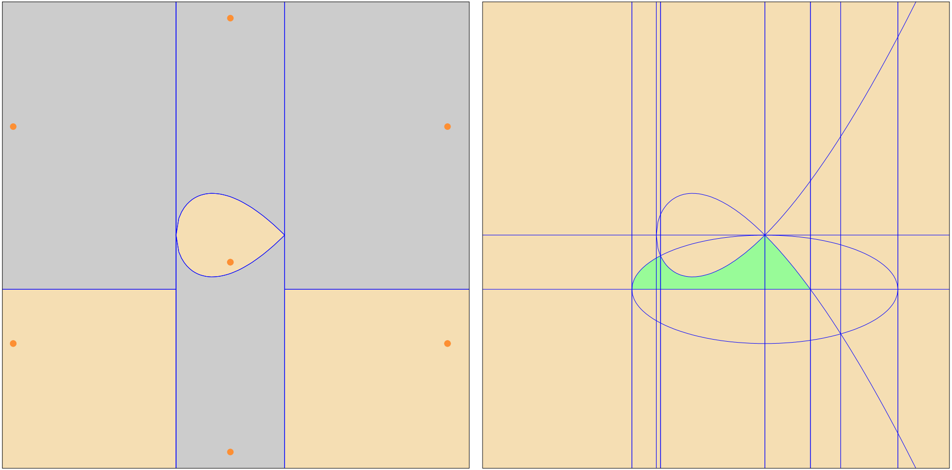

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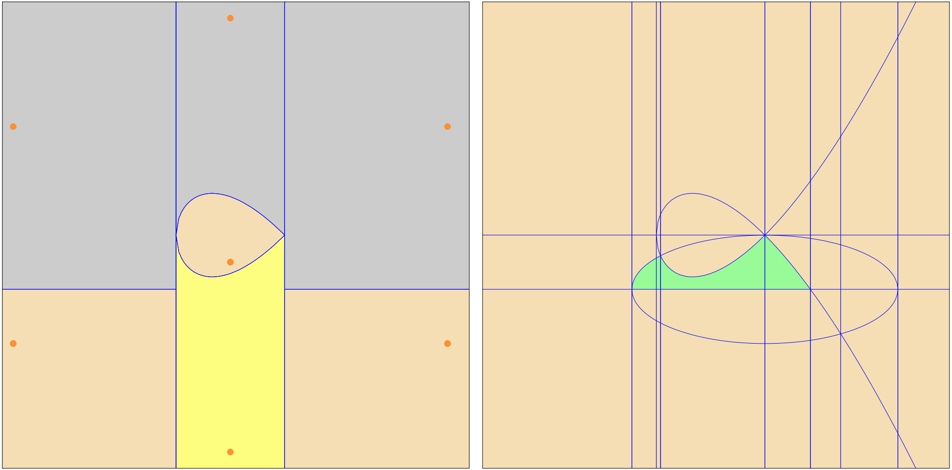

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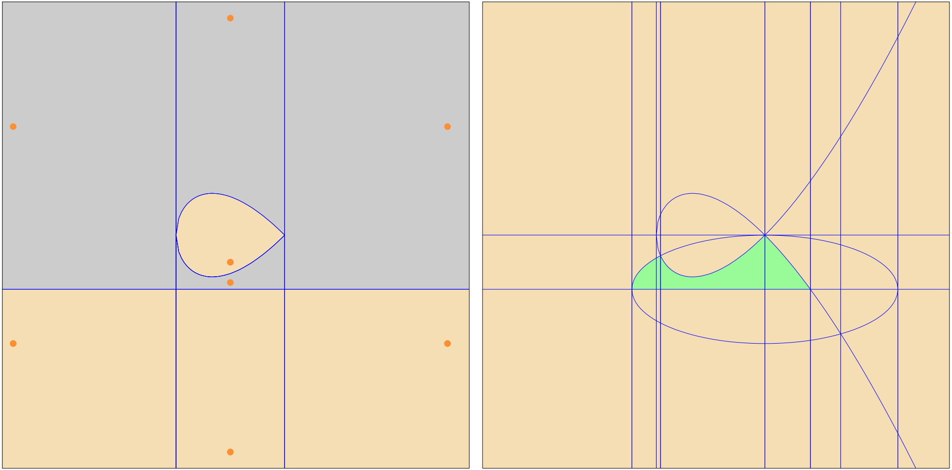

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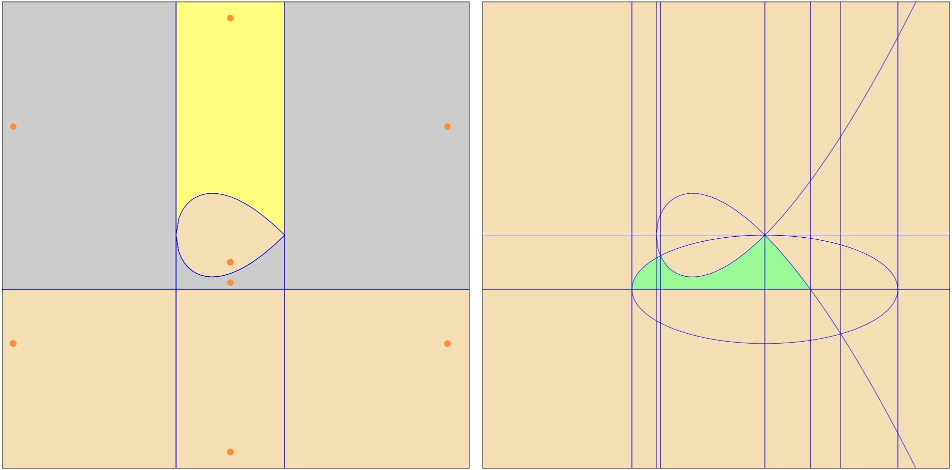

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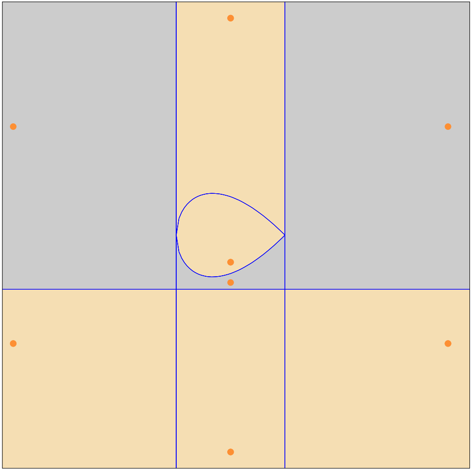

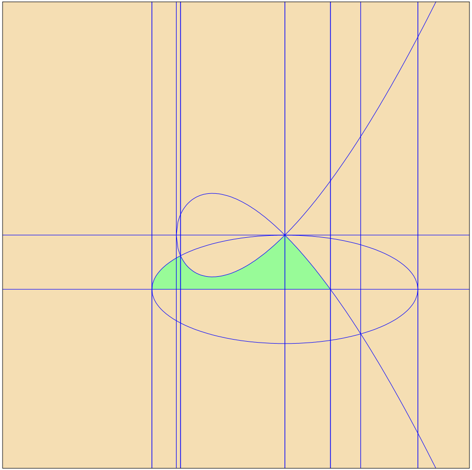

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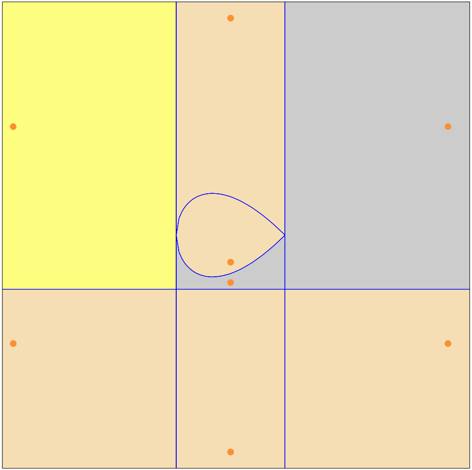

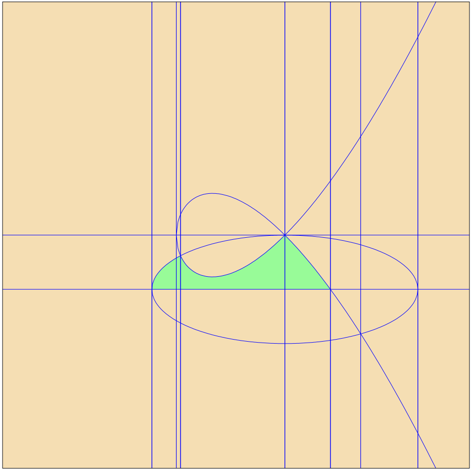

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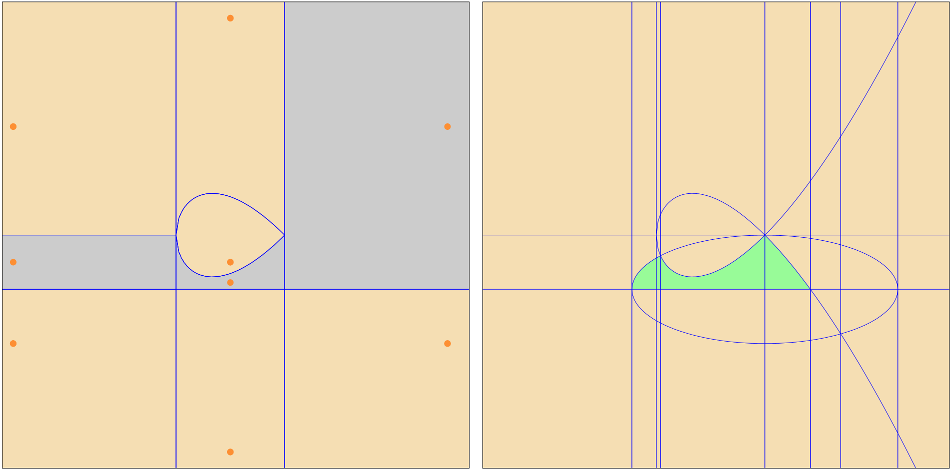

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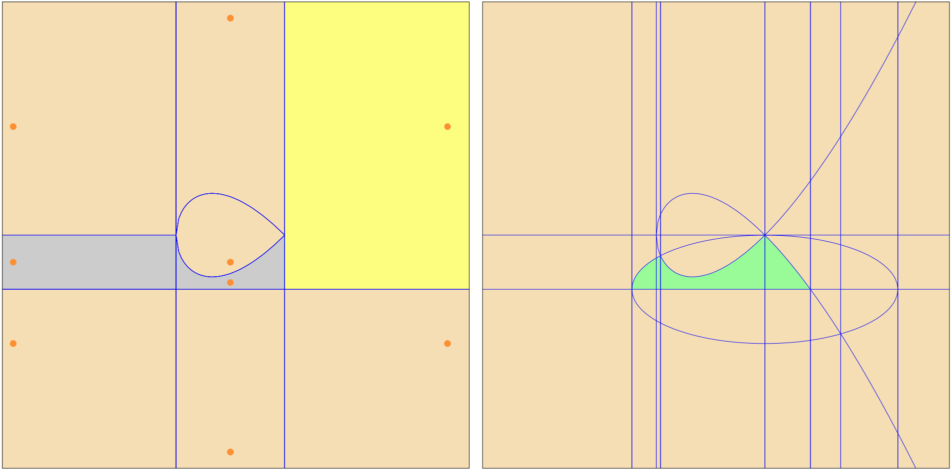

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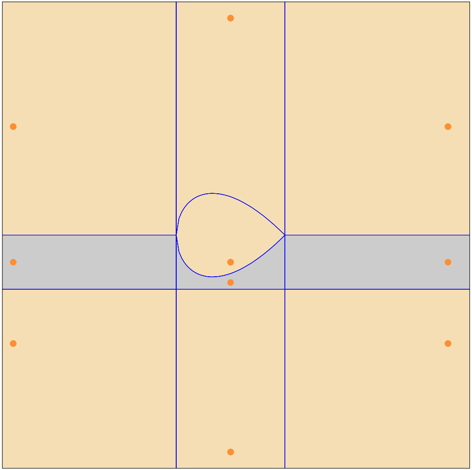

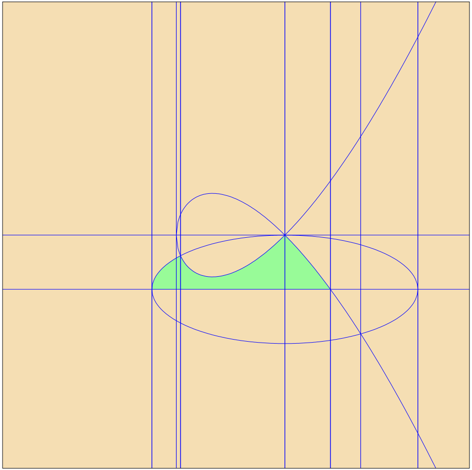

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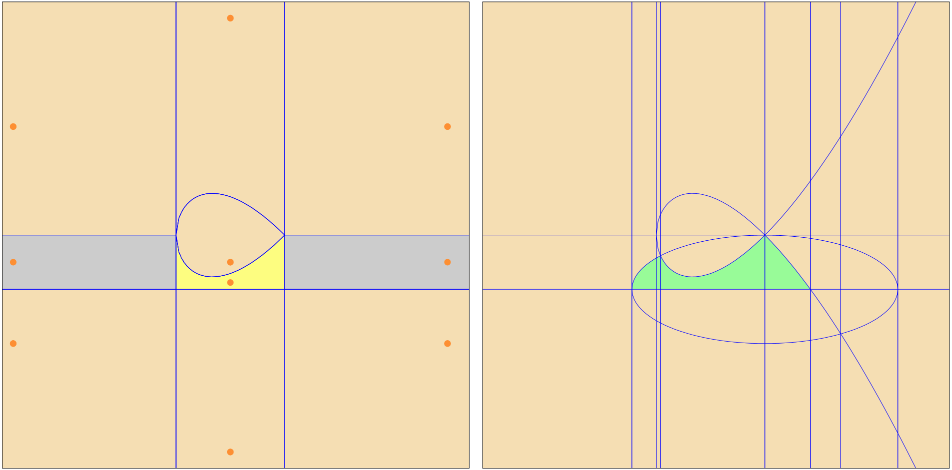

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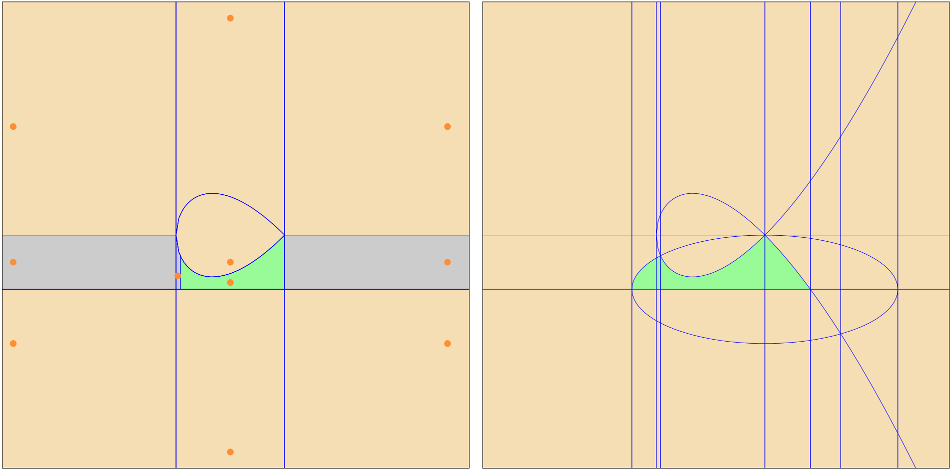

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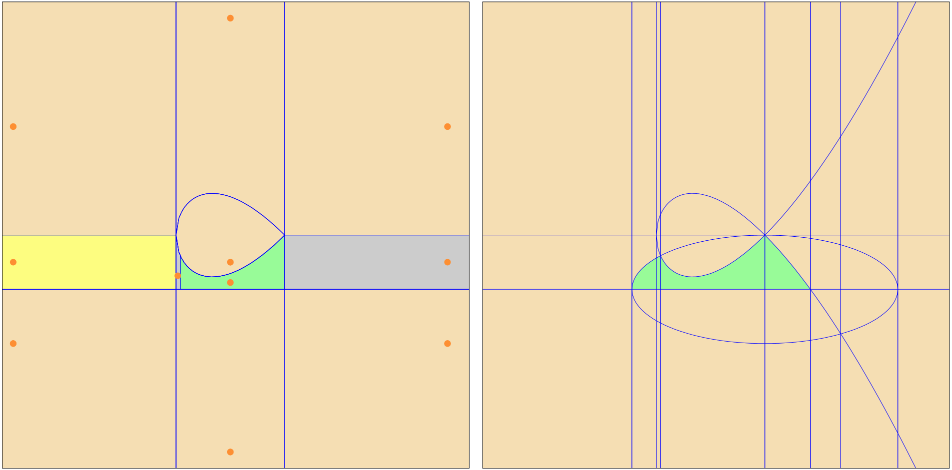

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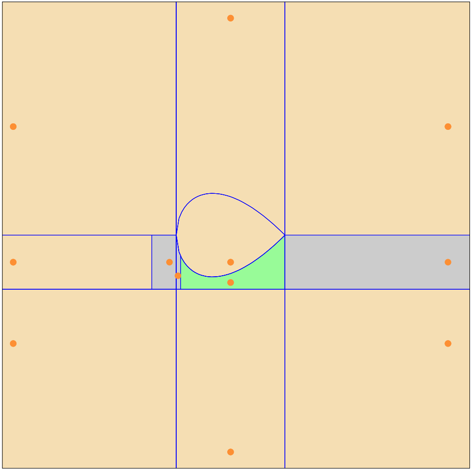

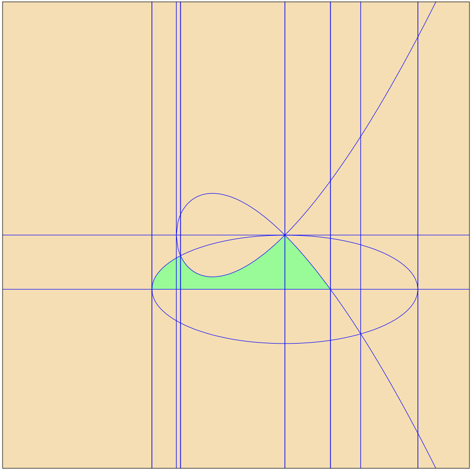

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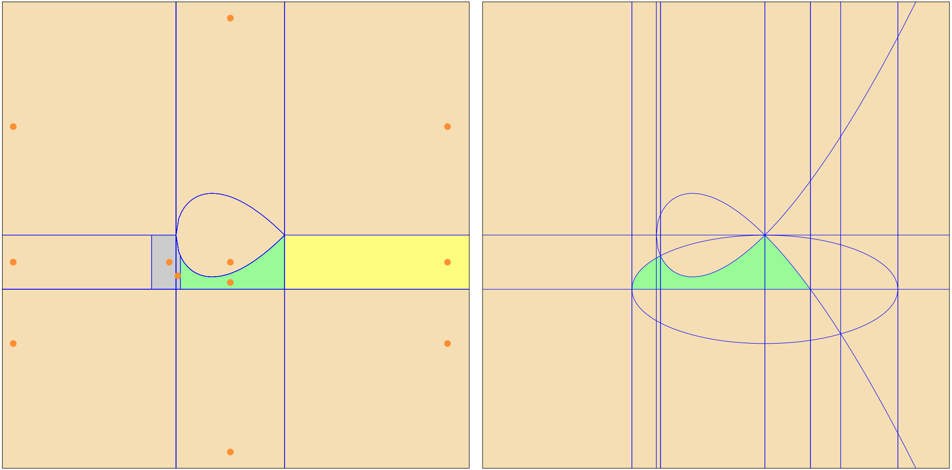

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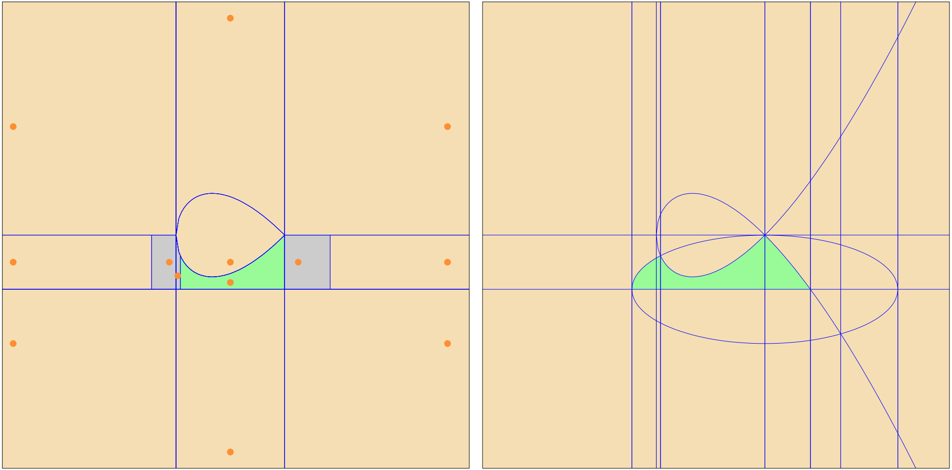

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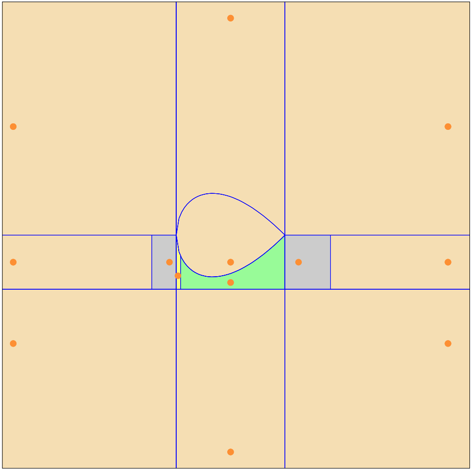

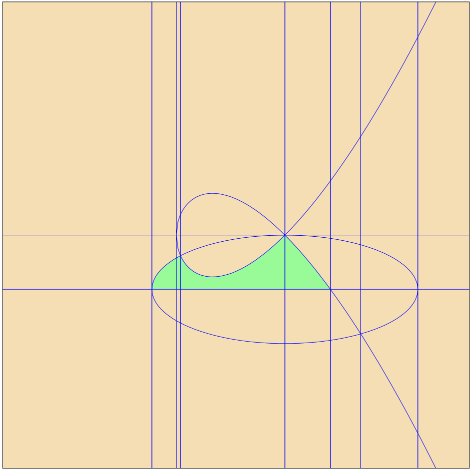

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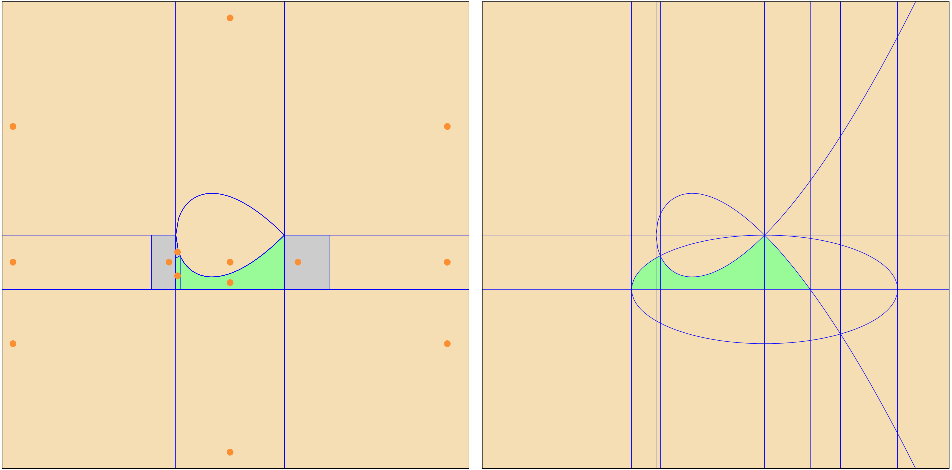

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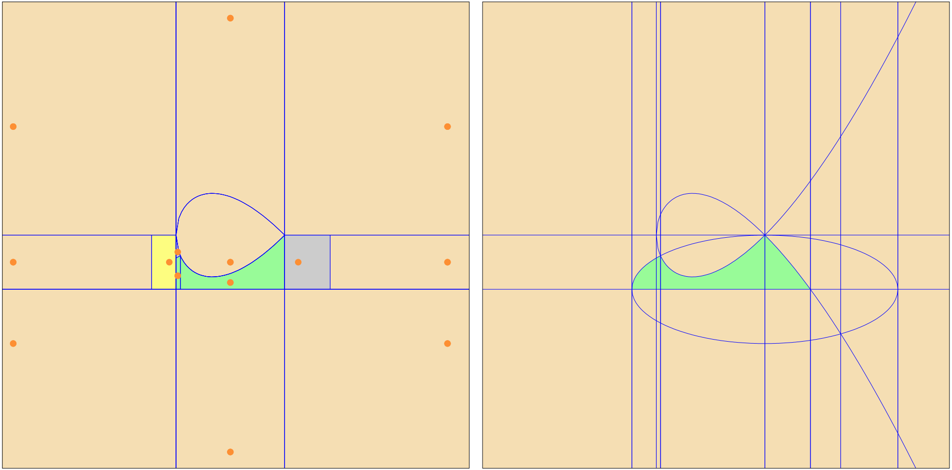

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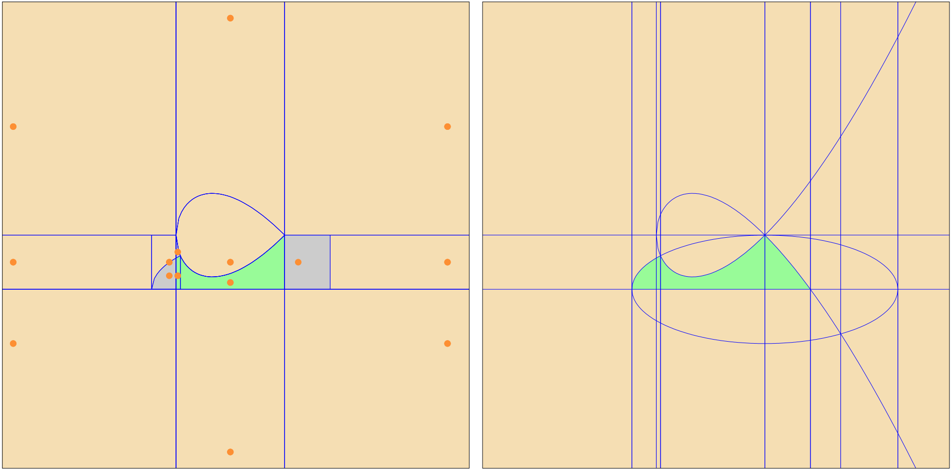

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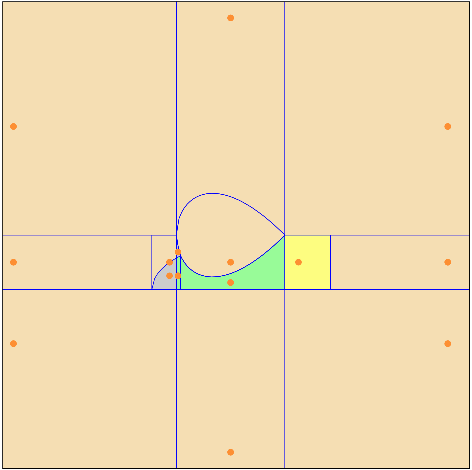

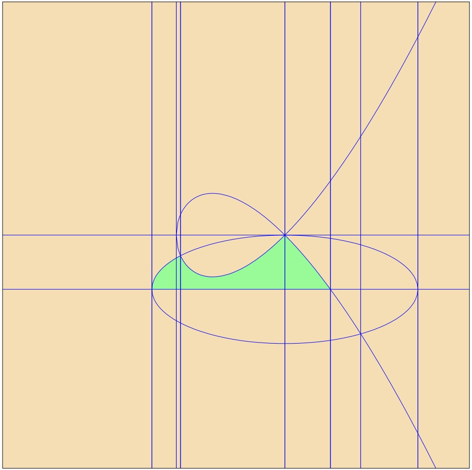

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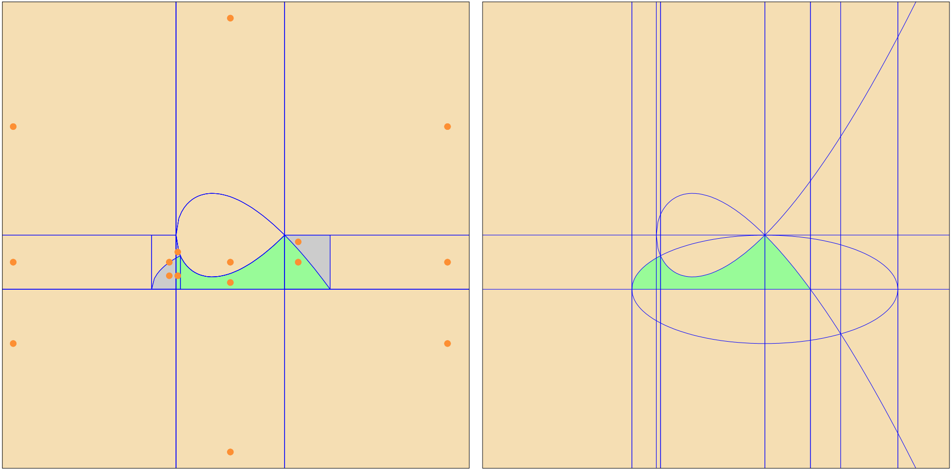

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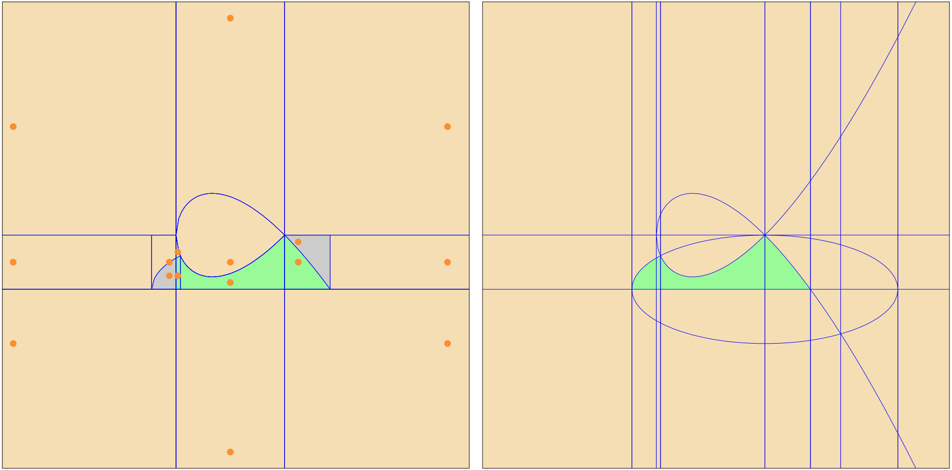

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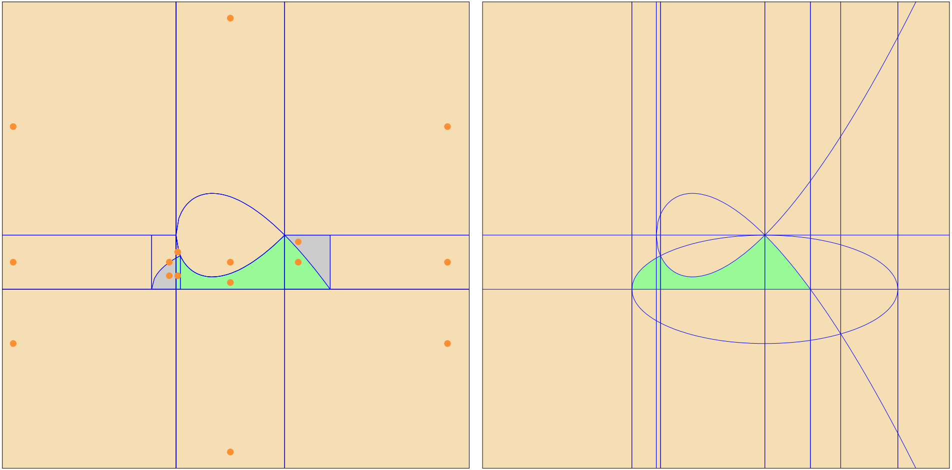

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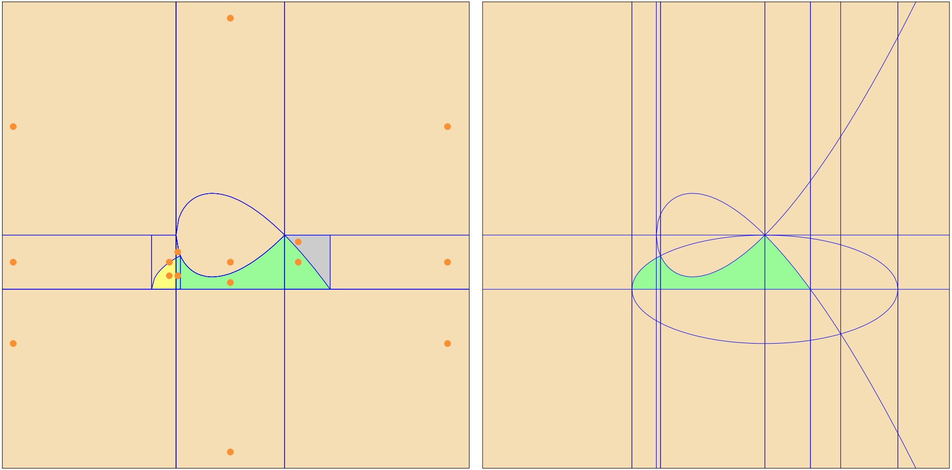

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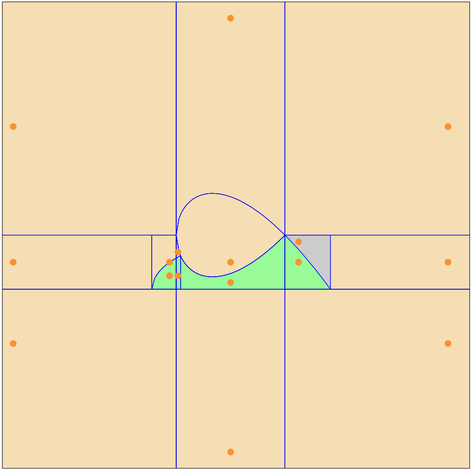

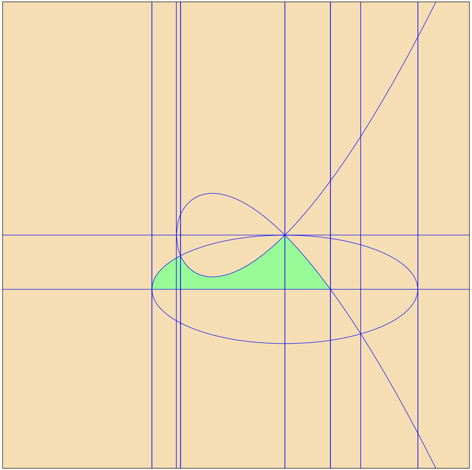

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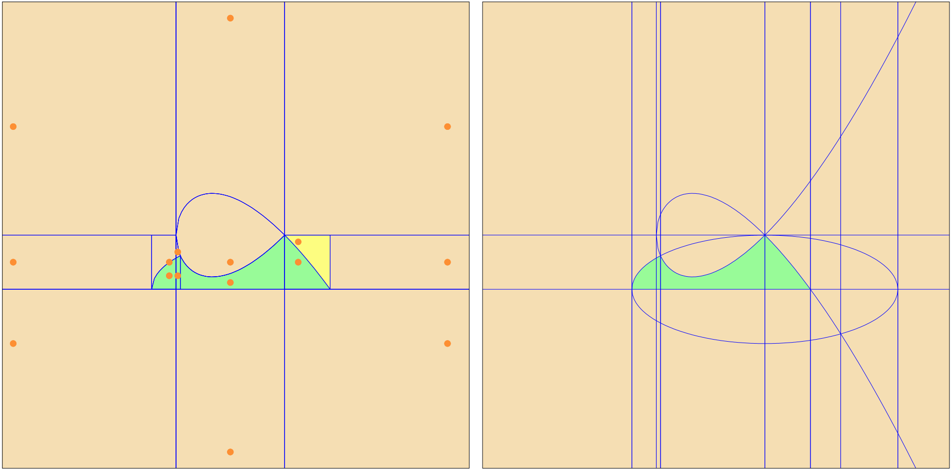

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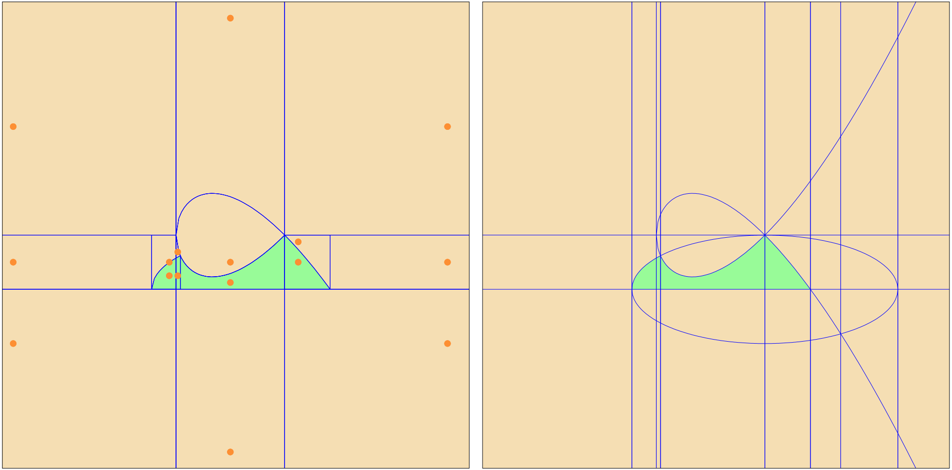

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Open NuCAD: an algorithm

Input: F, a Tarski formula in variables $x_1 \ldots, x_n$
Output: D, an Open NuCAD of $\mathbb{R}^n$ representing the set defined by F

Step 0: Q an empty queue

Step 1: enqueue cell $\mathbb{R}^n$ with sample point $\overline{0}$

Step 2: while Q not empty do

1. dequeue cell c with sample point α from Q
2. evaluate F at α, and choose subset S of polynomials in F whose sign conditions at α suffice to determine the T/F value of F
3. $c' = \text{merge}(c, \alpha, S)$ (add c' to the output D)
4. enqueue cells comprising $c - c'$

Experimental Results: linear input

- compare the number of cells in Open CAD vs. Open NuCAD
- conjunctions of positivity conditions on k random dense linear polynomials in $x_1, \ldots, x_n$
- average over 25 instances

Linear Set of Experiments			
	4 polys	5 polys	6 polys
2 vars	5.6 35.0	6.9 64.0	5.6 104.6
3 vars	21.9 271.5	24.0 1716.0	38.7 7392.0
4 vars	41.0 525.2	233.1 249149.0	582.2 $\approx$ 8500000.0

Experimental Results: non-linear input

- compare the number of cells in Open CAD vs. Open NuCAD
- conjunctions of positivity conditions on k random dense barely non-linear polynomials in $x_1, \ldots, x_n$
- average over 25 instances

Non-linear Set of Experiments			
vars	4 polys	5 polys	6 polys
2	9.6 59.5	10.9 111.8	11.5 185.0
3	128.4 2079.7	163.1 8535.3	176.6 30736.0
4	4147.1 446927.8	8403.4 $>$ 8000000.0	19466.8 $>$ 25000000.0

The Future

- full implementation (beyond conjunctions)
- extend to construct full decompositions
- theoretical analysis
- quantifier elimination by NuCAD
- explore optimizations